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Solving the flavour problem with hierarchical fermion wave functions

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ABSTRACT

We investigate the flavour structure of generic extensions of the SM where quark and lepton mass hierarchies and the suppression of flavour-changing transitions originate only by the normalization constants of the fermion kinetic terms. We show that in such scenarios the contributions to quark FCNC transitions from dimension-six effective operators are sufficiently suppressed without (or with modest) fine tuning in the effective scale of new physics. The most serious challenge to this type of scenarios appears in the lepton sector, thanks to the stringent bounds on LFV. The phenomenological consequences of this scenario in view of improved experimental data on quark and lepton FCNC transitions, and its differences with respect to the Minimal Flavour Violation hypothesis are also discussed.

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1. Introduction

The Standard Model (SM) can be regarded as the low-energy limit of a general effective Lagrangian. Within the renormalizable part of such a Lagrangian there are two types of operators involving fermion fields: kinetic and Yukawa terms. In general both these terms can have a non-trivial flavour structure. Since the description of physics is invariant under field reparametrizations, it is only the relative flavour structure between kinetic and Yukawa terms that has a physical meaning.

Employing the canonical normalization of the kinetic terms, the physical flavour structure of the SM (or the renormalizable part of the effective theory) is manifest in the Yukawa matrices. As it is well known, these have a quite peculiar form: their eigenvalues are very hierarchical and the two matrices in the quark sector are quasi-aligned in flavour space, with the misalignment parameterized by the CKM matrix. This structure is responsible for the great successes of the SM in the flavour sector, including the strong suppression of CP-violating and flavour-changing neutral current (FCNC) amplitudes.

Several additional flavour structures could appear in the tower of higher-dimensional operators which belongs to the non-renormalizable part of the effective Lagrangian. However, if we assume an effective scale of new physics in the TeV range (as expected by a natural stabilization of the electro-weak symmetry-breaking

sector), present experimental results leave a very limited room for new flavour structures. A natural way out to this problem, the so-called flavour problem, is provided by the hypothesis of minimal flavour violation (MFV) [1–3]. According to this hypothesis, there exists a flavour symmetry defined by canonically normalized kinetic terms, and the Yukawa matrices are its only breaking sources. In other words, the SM Yukawa matrices are treated as the only non-negligible spurions of the $SU(3)^5$ flavour symmetry [2].

The MFV hypothesis is a simple and effective solution to the flavour problem [4,5], but is far from being the only allowed possibility. Various alternatives or variations of this hypothesis have indeed been proposed in the recent literature [6–10]. The question we would like to address in this work is the viability, in general terms, of solutions to the flavour problem based not on a flavour symmetry in the low-energy effective theory, but instead on hierarchies in the kinetic terms which suppress flavour-changing transitions. This could be a “democratic” alternative, where the many coupling constant matrices can be of any form, and the only restriction is that the kinetic terms should have a hierarchical normalization. The wave function normalization factors then function as a distorting lense, through which all interactions are seen as approximately aligned on the magnification axes of the lense.

From a model-building point of view, hierarchical fermion wave functions (and corresponding hierarchical kinetic terms) can emerge in scenarios with extra dimensions, where the hierarchy in the four-dimensional wave functions reflect their non-trivial profile in the extra dimension [11,12]. In particular, a suppression of this type can be present in Randall–Sundrum (RS) scenarios [13], whose flavour phenomenology was studied in Refs. [14–16]. In the recent literature, variations of such models have been constructed

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with particular attention to the flavour structure [17–19].¹ Hierarchical fermion wave functions can also arise due to Renormalisation Group running, with large, positive, and distinct anomalous dimensions for the standard model fields of different generations [20].

It is always possible to redefine fields so as to choose a basis in which the Yukawa interactions are not hierarchical, and the SM flavour structure manifests itself through the hierarchy of the kinetic terms. As long as we look only at the renormalizable part of the low-energy effective Lagrangian, there is no way to isolate the origin of the flavour hierarchy (kinetic vs. Yukawa terms). However, the different assumptions about its origin lead to different ansätze for the flavour structure of the higher-dimensional operators. In this Letter we analyse the class of scenarios where dimensionless coupling matrices (both Yukawas and non-renormalizable operators) have $\mathcal{O}(1)$ entries. That is, they do not exhibit a specific flavour structure in a basis where the kinetic terms are hierarchical. More explicitly, we investigate whether it is possible to choose a hierarchy for the fermion wave functions such that, after moving to the canonical basis, the contributions from dimension-six FCNC operators are sufficiently suppressed and the Standard Model Yukawa hierarchies are obtained. We deem the scenario to work if the dimensional suppression scale of the operators turns out to be $\lesssim 10$ TeV (as in the MFV scenario). This corresponds to one-loop contributions from new particles of mass $\sim 3m_Z$ and SM gauge couplings. We analyse this problem both in the quark and in the lepton sector, and we discuss the differences arising with respect to the MFV scenario.

The scenario we are considering has some similarities with the Next-to-MFV framework of Ref. [6], where flavoured couplings induced by New Physics (NP) are “quasi-aligned” to the SM Yukawas, and where the NP couples “dominantly” to the third generation. The hierarchical wave-function scenario is an example of how New Physics could interact dominantly with the third generation; however, this scenario differ from the NMFV hypothesis studied in Ref. [6]. There, the Lagrangian has a $U(2)^3$ symmetry acting on the quarks of the first two generations (broken only by the quark Yukawas) and new physics interactions involving the third generation are arbitrary. The hierarchical wave functions we consider differ by not appealing to any flavour symmetry, or to a restricted set of spurions. This difference has non-negligible phenomenological implications in the case of rare transitions among the first two generations of quarks and leptons. Yet another definition of “dominant coupling to the third generation” is used by the UTfit Collaboration [5], who take $\Delta F = 2$ operators of arbitrary chirality to all have the same CKM-like suppression $\propto |V_{ti}^* V_{tj}|^2$. Our bounds differ also from those in Ref. [5], especially in the case of chirality-flipping operators, although in practice we find similar conclusions about the key role of ϵ_K in constraining both their and our scenario.

The Letter is organized as follows: in Section 2 we discuss the basic setup of our scenario for the quark sector. The corresponding bounds on the effective operators from quark FCNCs are analysed in Section 3. The extension to the lepton sector and the bounds from lepton FCNC transitions are discussed in Section 4. Section 5 is devoted to a general discussion of the bounds and a comparison

with the MFV scenario. The results are summarized in the conclusions.

2. Basic setup for the quark sector

The operators involving quark fields in the renormalizable part of the effective Lagrangian are

$$\mathcal{L}_{\text{quarks}}^{d=4} = \overline{Q}_L X_Q \not{D} Q_L + \overline{D}_R X_D \not{D} D_R + \overline{U}_R X_U \not{D} U_R + \overline{Q}_L Y_D D_R H + \overline{Q}_L Y_U U_R H_c, \quad (1)$$

where $H_c = i\tau_2 H^*$. In a generic non-canonical basis, the three $X_{Q,D,U}$ and the two $Y_{D,U}$ are 3×3 complex matrices. The usual choice of field normalization and basis is obtained by diagonalising the kinetic terms, re-scaling the fields to obtain the canonical normalization of the kinetic terms, and finally diagonalising the down-quark masses. With this choice

$$X_Q = X_D = X_U = I, \quad Y_D = \lambda_D, \quad Y_U = V^\dagger \lambda_U, \quad (2)$$

where I is the identity matrix, V is the CKM matrix and

$$\lambda_D = \frac{1}{v} \text{diag}(m_d, m_s, m_b), \quad \lambda_U = \frac{1}{v} \text{diag}(m_u, m_c, m_t), \quad (3)$$

with $v = (\langle H^\dagger H \rangle)^{1/2} = 174$ GeV.

For the purposes of this Letter we express the $d = 4$ Lagrangian with a different, hierarchical, field normalization for the kinetic terms, and then add the higher-dimensional operators with “democratic” flavour couplings. An example of the bases where the Yukawas are no longer hierarchical can be reached starting from the basis (2), by performing a unitary transformation on U_R and by an appropriate rescaling of the fields. In particular, denoting by Z_A ($A = Q, U, D$) the diagonal matrices by which we rescale the fermion fields ($Q_A \rightarrow Z_A^{-1} Q_A$) and by W_A the complex matrices describing their unitary transformations in the canonical basis ($W_U^\dagger W_U = I$), we can move to a non-canonical basis where²

$$Y_D \rightarrow Z_Q^{-1} \lambda_D Z_D^{-1} = I, \quad (I)_{ij} = \delta_{ij}, \quad (4)$$

$$Y_U \rightarrow Z_Q^{-1} V^\dagger \lambda_U W_U Z_U^{-1} = T_U \quad (T_U)_{ij} = \mathcal{O}(1). \quad (5)$$

In this new basis the hierarchical structure usually attributed to the Yukawa couplings is hidden in the flavour structure of the (diagonal) kinetic terms:

$$X_Q = Z_Q^{-2}, \quad X_D = Z_D^{-2}, \quad X_U = Z_U^{-2}. \quad (6)$$

The conditions we have imposed on the Z_A in Eqs. (4) and (5) do not specify completely their structure. However, assuming the maximal entries in the Z_A are at most of $\mathcal{O}(1)$ implies a hierarchical structure of the type

$$Z_A = \text{diag}(z_A^{(1)}, z_A^{(2)}, z_A^{(3)}), \quad z_A^{(1)} \ll z_A^{(2)} \ll z_A^{(3)} \lesssim 1. \quad (7)$$

The framework we want to investigate is a scenario where the hierarchical structure in Eq. (7) is the only responsible for the natural suppression of FCNCs. More explicitly, we assume that with hierarchical normalization of the kinetic terms, Eq. (6), all the higher-dimensional operators of the effective Lagrangian have a generic $\mathcal{O}(1)$ structure, such as the up-type Yukawa coupling in Eq. (5). In this framework the suppression of FCNCs arise by the rescaling the fermion fields necessary for the canonical normalization of the kinetic terms:

$$Q_A \rightarrow Z_A Q_A. \quad (8)$$

¹ The scenario of Ref. [17], where the suppression of flavour-changing transitions arises only by the mixing of the SM fermions with the composite fields (in the dual four-dimensional description of a five-dimensional warped geometry) is a clear example of the class of models we intend to study. Although the scenario we consider emerges naturally in the RS framework, other solutions to the flavour problem have been proposed in the RS context. For instance, additional symmetries can be imposed in warped geometry in order to recover a MFV structure in four [18] or five dimensions [19].

² With different unitary transformation we could have chosen $Y_U \rightarrow I$ and $Y_D \rightarrow \mathcal{O}(1)$, or $Y_{U,D} \rightarrow \mathcal{O}(1)$. As it will become clear in the next section, the choice in Eqs. (4)–(5), is the simplest one for the phenomenological analysis of FCNC constraints.

Starting from non-hierarchical bilinear structures,

$$\bar{A}X_{AB}B, \quad X_{AB}^{ij} = \mathcal{O}(1), \quad (9)$$

the transformation into the canonical basis move the Z_A into the effective couplings, with a corresponding suppression of the flavour-changing terms:

$$X_{AB}^{ij} \rightarrow (Z_A X_{AB} Z_B)^{ij} \sim Z_A^{(i)} Z_B^{(j)}. \quad (10)$$

This illustrates a difference between hierarchical wave functions, and Froggatt–Nielsen [21] type models [22–24]. Both may reproduce the observed Yukawa hierarchy, but a simple Froggatt–Nielsen model [24], where $Z_A^{(i)} \sim \epsilon^{Q_A^i - Q_B^j}$ would give less suppression of FCNC processes: $X_{AB}^{ij} \rightarrow \epsilon^{|Q_A^i - Q_B^j|}$.

In the next section we analyse which conditions the Z_A should satisfy in order to provide a suppressions of FCNCs compatible with experimental data, while keeping the effective scale of New Physics in the TeV range. As we will show, in the quark sector these conditions are compatible with Eqs. (4)–(5), namely with a natural generation of the observed Yukawa couplings starting from generic $\mathcal{O}(1)$ structures. The only exceptions are the kaon constraints from ϵ_K and ϵ'/ϵ , which however can be fulfilled with a modest fine tuning ($\sim 10^{-1}$) in the coupling of the effective operators.

3. Bounds from quark FCNCs

3.1. $\Delta F = 2$ operators

As a first step we would like to constrain the parameters introduced in (7) with the help of processes involving $\Delta F = 2$ operators.

There are in principle eight dimension-six four-quark operators that can contribute to down-type $\Delta F = 2$ processes [25]. However, restricting the attention to operators which preserve the $SU(2)_L \otimes U(1)_Y$ gauge symmetry, this number reduces to four:

$$\begin{aligned} O_{LL}^{ij} &= \frac{1}{2} (\bar{Q}_L^i \gamma_\mu Q_L^j)^2, & O_{LR1}^{ij} &= (\bar{Q}_L^i \gamma_\mu Q_L^j) (\bar{D}_R^i \gamma_\mu D_R^j), \\ O_{RR}^{ij} &= \frac{1}{2} (\bar{D}_R^i \gamma_\mu D_R^j)^2, & O_{LR2}^{ij} &= (\bar{D}_R^i Q_L^j) (\bar{Q}_L^i D_R^j). \end{aligned} \quad (11)$$

In order to derive model-independent bounds on the coupling of these operators, we introduce the following effective Hamiltonian (defined at the electroweak scale with canonically-normalized fields):

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \left(C_{\text{SM}}^{ij} + \frac{X_{LL}^{ij}}{\Lambda^2} \right) O_{LL}^{ij} + \frac{X_{RR}^{ij}}{\Lambda^2} O_{RR}^{ij} + \frac{X_{LR1}^{ij}}{\Lambda^2} O_{LR1}^{ij} + \frac{X_{LR2}^{ij}}{\Lambda^2} O_{LR2}^{ij} + \text{h.c.} \\ &= C_{LL}^{ij}(M_W) O_{LL}^{ij} + C_{RR}^{ij}(M_W) O_{RR}^{ij} + C_{LR1}^{ij}(M_W) O_{LR1}^{ij} \\ &\quad + C_{LR2}^{ij}(M_W) O_{LR2}^{ij} + \text{h.c.}, \end{aligned} \quad (12)$$

where

$$C_{\text{SM}}^{ij} = \frac{G_F^2}{2\pi^2} M_W^2 (V_{ti}^* V_{tj})^2 S_0(x_t). \quad (13)$$

The amplitudes of the various $\Delta F = 2$ processes can be calculated renormalizing the effective Hamiltonian (12) at the relevant low scale μ (e.g. $\mu \approx m_b$ for $\bar{B}^0 - B^0$ mixing). The RGE running of the Wilson coefficients can be written as

$$\begin{pmatrix} C_{LL}^{ij}(\mu) \\ C_{RR}^{ij}(\mu) \\ \tilde{C}_{LR}^{ij}(\mu) \end{pmatrix} = \begin{pmatrix} \eta_{LL}^{ij}(\mu, M_W) & & \\ & \eta_{RR}^{ij}(\mu, M_W) & \\ & & \hat{\eta}_{LR}^{ij}(\mu, M_W) \end{pmatrix} \times \begin{pmatrix} C_{LL}^{ij}(M_W) \\ C_{RR}^{ij}(M_W) \\ \tilde{C}_{LR}^{ij}(M_W) \end{pmatrix}. \quad (14)$$

Since QCD preserves chirality, there is no mixing between the LL , RR and LR sectors and $\eta_{LL}^{ij} = \eta_{RR}^{ij}$. The only non-trivial mixing occurs among the two LR operators, where $\hat{\eta}_{LR}^{ij}(\mu, M_W)$ is a 2×2 matrix and

$$\tilde{C}_{LR}^{ij} = \begin{pmatrix} C_{LR1}^{ij}(\mu) \\ C_{LR2}^{ij}(\mu) \end{pmatrix} = \hat{\eta}_{LR}^{ij}(\mu, M_W) \begin{pmatrix} C_{LR1}^{ij}(M_W) \\ C_{LR2}^{ij}(M_W) \end{pmatrix}. \quad (15)$$

The analytic formulae for these RGE factors as well as the relevant hadronic matrix elements can be found in Ref. [25]. Following the approach given therein, we can express the $\Delta F = 2$ amplitude for a generic neutral meson mixing as

$$\mathcal{A}^{ij} = \langle \bar{M}_{ij}^0 | \mathcal{H}_{\text{eff}}(\mu) | M_{ij}^0 \rangle \propto P_{LL}(C_{LL} + C_{RR}) + P_{LR1} C_{LR1} + P_{LR2} C_{LR2}, \quad (16)$$

where the P_A factors are appropriate combinations of RGE coefficients and hadronic matrix elements. Experimentally we have several precise constraints on this type of amplitudes: both $|\mathcal{A}^{31}|$ and $\arg(\mathcal{A}^{31})$ are constrained by ΔM_{B_d} and $S_{\psi K_S}$; $|\mathcal{A}^{32}|$ is constrained by ΔM_{B_s} ; $\text{Im}(\mathcal{A}^{12})$ is constrained by ϵ_K . In the following we will impose that the non-standard contribution is within $\pm 10\%$ of the SM contribution, in magnitude, for all down-type $\Delta F = 2$ mixing amplitudes.

Let us first analyze the LL sector. Here the situation is quite simple since we can factorise the new-physics contribution as a correction to the SM Wilson coefficient:

$$\begin{aligned} \mathcal{A}^{ij}|_{LL} &= \mathcal{A}_{\text{SM}}^{ij} \left(1 + \frac{X_{LL}^{ij}}{(V_{ti}^* V_{tj})^2} \frac{F^2}{\Lambda^2} \right), \\ F &= \left(\frac{2\pi^2}{G_F^2 M_W^2 S_0(x_t)} \right)^{\frac{1}{2}} \approx 3 \text{ TeV}. \end{aligned} \quad (17)$$

Since the effective scale of the SM contribution is 3 TeV, if new physics contributes via loops and is weakly interacting (as the electroweak SM contribution), taking $\Lambda \sim 10$ TeV corresponds to new masses of the order of $3M_Z$. Allowing for the amplitude (17) to vary from its SM values by at most $\pm 10\%$, in magnitude, lead to

$$\sqrt{|X_{LL}^{ij}|} < 0.3 |V_{ti}^* V_{tj}| \left(\frac{\Lambda}{F} \right) < |V_{ti}| |V_{tj}| \quad \text{for } \Lambda < 10 \text{ TeV}. \quad (18)$$

Expressing the flavour structure of the X_{LL}^{ij} in terms of the corresponding hierarchical fermion wave functions, as in Eqs. (8)–(10), we find

$$\sqrt{|X_{LL}^{ij}|} \sim |z_Q^{(i)} z_Q^{(j)}| < |V_{ti}| |V_{tj}| \rightarrow |z_Q^{(i)}| < |V_{ti}|. \quad (19)$$

Since $\eta_{LL} = \eta_{RR}$ and $\langle O_{LL}^{ij} \rangle = \langle O_{RR}^{ij} \rangle$, the constraint on the RR operator is completely analog to the LL one:

$$\sqrt{|X_{RR}^{ij}|} \sim |z_D^{(i)} z_D^{(j)}| < |V_{ti}| |V_{tj}| \rightarrow |z_D^{(i)}| < |V_{ti}|. \quad (20)$$

In the LR sector the situation is slightly more complicated because of the different matrix elements involved. The P_A factors introduced in Eq. (16) can be decomposed as [25]:

$$P_{LL} = \eta_{LL}(\mu, M_W) B_{LL}(\mu), \quad (21)$$

$$\begin{aligned} P_{LR1} &= -\hat{\eta}_{LR}(\mu, M_W)_{11} [B_{LR1}(\mu)]_{\text{eff}} \\ &\quad + \frac{3}{2} \hat{\eta}_{LR}(\mu, M_W)_{21} [B_{LR2}(\mu)]_{\text{eff}}, \end{aligned} \quad (22)$$

$$\begin{aligned} P_{LR2} &= -\hat{\eta}_{LR}(\mu, M_W)_{12} [B_{LR1}(\mu)]_{\text{eff}} \\ &\quad + \frac{3}{2} \hat{\eta}_{LR}(\mu, M_W)_{22} [B_{LR2}(\mu)]_{\text{eff}}. \end{aligned} \quad (23)$$

In the specific case of $\bar{B}^0 - B^0$ we can further write

$$\eta_{LL}(\mu, M_W) B_{LL}(\mu) = \eta_B \hat{B}_{B_q}, \quad (24)$$

$$[B_{LRi}(\mu)]_{\text{eff}} = \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 B_{LRi}(\mu), \quad (25)$$

where \hat{B}_{B_q} is the RGE invariant bag factor of the SM (LL) operator. Using the RGE factors in Refs. [25,26] and the $B_{LRi}(\mu)$ factors from lattice [5,27] leads to

$$P_{LL} = 0.7, \quad P_{LR1} = -5.0, \quad P_{LR2} = 6.3, \quad (26)$$

with negligible differences between B_s and B_d cases. The contributions of the $O_{LR1(2)}$ operator is thus enhanced by a factor $|P_{LR1(2)}/P_{LL}| \approx 7(9)$ compared to the SM one. Allowing at most $\pm 10\%$ corrections to the SM amplitude, the bounds derived from B_s and B_d mixing are

$$|X_{LR1}^{3j}| \sim |z_Q^{(3)} z_Q^{(j)} z_D^{(3)} z_D^{(j)}| < 0.2 |V_{tj}|^2, \quad (27)$$

$$|X_{LR2}^{3j}| \sim |z_D^{(3)} z_Q^{(j)} z_Q^{(3)} z_D^{(j)}| < 0.1 |V_{tj}|^2, \quad (28)$$

where we have set $V_{tb} = 1$ and $\Lambda < 10$ TeV. These inequalities are satisfied, with λ_D^{ii} evaluated at m_W , if we assume the hierarchy

$$z_Q^{(i)} z_D^{(i)} = (\lambda_D)_{ii}, \quad (29)$$

which follows from the definition of $z_Q^{(i)}$ and $z_D^{(i)}$ in Eq. (4).

Matrix elements and RGE factors lead to $P_{LR1(2)}$ substantially larger in the case of $\bar{K}^0 - K^0$ mixing [25]:

$$P_{LL} = 0.5, \quad P_{LR1} = -0.7 \times 10^2, \quad P_{LR2} = 1.1 \times 10^2. \quad (30)$$

Proceeding as in the $\bar{B}^0 - B^0$ case, this implies the stringent bound

$$|X_{LR2}^{21}| \sim |z_Q^{(2)} z_Q^{(1)} z_D^{(2)} z_D^{(1)}| < 0.004 |V_{ts}^* V_{td}|^2 \approx 0.6 \times 10^{-9}, \quad (31)$$

and similarly for $|X_{LR1}^{21}|$. In such case, using Eq. (29) the bound is not fulfilled by about one order of magnitude:³

$$|z_Q^{(2)} z_Q^{(1)} z_D^{(2)} z_D^{(1)}| \sim \frac{m_d m_s}{v^2} \approx 1 \times 10^{-8}. \quad (32)$$

This result is qualitatively similar to the conclusion of Ref. [5], where ϵ_K has been identified as the most significant $\Delta F = 2$ constraint on (non-MFV) models where NP couples dominantly to the third generation.

3.2. $\Delta F = 1$ operators

Taking into account the analysis of the $\Delta F = 2$ sector, it is clear that $\Delta F = 1$ operators with a LL (or RR) structure, such as $\bar{Q}_L^i \gamma^\mu Q_L^j H^\dagger D_\mu H$ do not represent a serious problem. The corresponding constraints are equivalent to those derived from LL and RR $\Delta F = 2$ operators, which are naturally fulfilled. On the other hand, a potentially interesting class of new constraints in the $\Delta F = 1$ sector arises by magnetic and chromomagnetic operators:

$$O_{RL\gamma}^{ij} = e H^\dagger \bar{D}_R^i \sigma^{\mu\nu} Q_L^j F_{\mu\nu}, \quad O_{RLg}^{ij} = g_s H^\dagger \bar{D}_R^i \sigma^{\mu\nu} T^a Q_L^j G_{\mu\nu}^a \quad (33)$$

In the case of the $O_{RL\gamma}^{Fij}$ operators the most significant constraint is derived from $B \rightarrow X_s \gamma$. The leading effective Hamiltonian at the electroweak scale can be written as

$$\mathcal{H}_{\text{eff}} = \left(C_{\text{SM}}^{32} + \frac{X_{RL\gamma}^{32}}{\Lambda^2} \right) O_{RL\gamma}^{32} + \frac{X_{RL\gamma}^{23}}{\Lambda^2} O_{RL\gamma}^{23} + \text{h.c.}, \quad (34)$$

³ Note that in the case of $\bar{K}^0 - K^0$ mixing we do not have a stringent experimental constraint on the modulo of the amplitude (given the large long-distance contributions to ΔM_K) but only on its imaginary part (thanks to ϵ_K): thus the order of magnitude disagreement concerns only the CPV part of the $\Delta S = 2$ amplitude.

where

$$C_{\text{SM}}^{32} = -\frac{G_F}{4\pi^2 \sqrt{2}} \lambda_b V_{ts}^* V_{td} C_7^{\text{SM}}(M_W), \quad (35)$$

and $C_7^{\text{SM}}(M_W^2) \approx -0.3$ is defined as in Ref. [28]. The contribution of $O_{RL\gamma}^{32}$ operator, which encodes also the SM contribution, can simply be taken into account by a shift of $C_7^{\text{SM}}(M_W^2)$ at the electroweak scale:

$$\frac{\delta C_7(M_W)}{C_7^{\text{SM}}(M_W)} = \frac{X_{RL\gamma}^{32}}{V_{ts}^* V_{tb}} \frac{1}{\lambda_b} \frac{F_B^2}{\Lambda^2}, \quad F_B = \left(-\frac{4\pi^2 \sqrt{2}}{C_7^{\text{SM}} G_F} \right)^{\frac{1}{2}} \approx 5 \text{ TeV}. \quad (36)$$

Using the approximate expression [28,29]

$$\frac{\mathcal{B}(B \rightarrow X_s \gamma)}{\mathcal{B}(B \rightarrow X_s \gamma)_{\text{SM}}} \approx 1 + 2.9 \times \delta C_7(M_W), \quad (37)$$

and allowing for a 15% departure from the SM value, leads to the bound (for $\Lambda < 10$ TeV):

$$|X_{RL\gamma}^{32}| \sim |z_D^{(3)} z_Q^{(2)}| < 0.5 |V_{ts}^* V_{tb}| \lambda_b \frac{\Lambda^2}{F_B^2} < 1.2 \times 10^{-3}. \quad (38)$$

Employing the hierarchy (29) and assuming $|z_Q^{(i)}| \sim |V_{ti}|$ (i.e. saturating the constraint (19) from $\Delta F = 2$ LL operators), this bound is naturally fulfilled:

$$|z_D^{(3)} z_Q^{(2)}| \sim |\lambda_b V_{ts}/V_{tb}| \approx 7 \times 10^{-4}. \quad (39)$$

Employing the same hierarchy, the coupling of the $O_{RL\gamma}^{23}$ operator is substantially larger: $|X_{RL\gamma}^{23}| \sim |z_D^{(2)} z_Q^{(3)}| \sim |\lambda_s V_{tb}/V_{ts}| \approx 8 \times 10^{-3}$. However, since this operator does not interfere with the SM contribution, the bound on $X_{RL\gamma}^{23}$ is weaker with respect to one in Eq. (38): $|X_{RL\gamma}^{23}| < 6 \times 10^{-3}$. We then conclude that the constraints from $B \rightarrow X_s \gamma$ are essentially fulfilled without fine tuning.

Similarly to the $\Delta F = 2$ sector, the most serious problems arise from K physics. Here the most significant constraints are obtained from the possible impact in ϵ'/ϵ of the chromomagnetic operators. The contribution of $\Delta I = 1/2$ operators (such as O_{RLg}^{12} and O_{RLg}^{21}) to ϵ'/ϵ can be generally written as

$$\delta \text{Re} \left(\frac{\epsilon'}{\epsilon} \right) \approx \frac{\omega}{\sqrt{2} |\epsilon| \text{Re} \mathcal{A}_0} \times \delta \text{Im} \mathcal{A}_0, \quad (40)$$

where $\mathcal{A}_I = \mathcal{A}(K^0 \rightarrow 2\pi_I)$ and $\omega = |A_2/A_0| \approx 1/22$. In the specific case of the chromomagnetic operators, following Refs. [30,31], we have

$$\begin{aligned} |\delta \text{Im} \mathcal{A}_0| &= \frac{|\text{Im}(X_{RLg}^{12} - X_{RLg}^{21})| v}{\Lambda^2} \eta_G (2\pi_{I=0} |g_s \bar{s}_R (\sigma \cdot G) d_L| K^0) \\ &= \frac{|\text{Im}(X_{RLg}^{12} - X_{RLg}^{21})|}{\Lambda^2} \eta_G B_G \sqrt{\frac{3}{2}} \frac{11}{2} \frac{m_\pi^2 m_K^2}{F_\pi \lambda_s}, \end{aligned} \quad (41)$$

where $X_{RLg}^{12(21)}$ are the couplings of the effective operators at the electroweak scale (defined in analogy to the $X_{RL\gamma}^{12(21)}$), η_G is the RGE factor, and B_G denote the bag parameter of the hadronic matrix element. Using the numerical values in Ref. [30] and imposing that the extra contribution to ϵ'/ϵ do not exceed 10^{-3} leads to the following bound:

$$\frac{|\text{Im}(X_{RLg}^{12} - X_{RLg}^{21})|}{\lambda_s} < 10^{-2} \left(\frac{\Lambda}{10 \text{ TeV}} \right)^2. \quad (42)$$

Using, as in the previous cases, the hierarchy (29) and $|z_Q^{(i)}| \sim |V_{ti}|$, we obtain

$$\left| \frac{X_{RLg}^{21}}{\lambda_s} \right| \sim \left| \frac{z_D^{(2)} z_Q^{(1)}}{\lambda_s} \right| \sim \left| \frac{V_{td}}{V_{ts}} \right| \approx 0.2,$$

$$\left| \frac{X_{RLg}^{12}}{\lambda_s} \right| \sim \left| \frac{z_D^{(1)} z_Q^{(2)}}{\lambda_s} \right| \sim \left| \frac{\lambda_d V_{ts}}{\lambda_s V_{td}} \right| \approx 0.3. \quad (43)$$

Similarly to the case of \bar{K}^0 – K^0 mixing, also in this case the suppression implied by the $z_A^{(i)}$ leads to a natural size about one order of magnitude larger than what is needed to fulfill the experimental constraints. In close analogy with the $\Delta S = 2$ case, the problem arises only from the imaginary (CP-violating) part of the amplitude.

4. Operators involving lepton fields

The approach we have followed so far in the quark sector can easily be exported also to the lepton sector. Introducing the diagonal matrices Z_L and Z_E , such that

$$Z_L^{-1} \lambda_E Z_E^{-1} \simeq I, \quad \lambda_E = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau), \quad (44)$$

we proceed investigating the impact of the transformation

$$L_L \rightarrow Z_L L_L, \quad E_R \rightarrow Z_E E_R, \quad (45)$$

onto operators involving lepton fields. A major difference with respect to the quark sector is that lepton flavour is conserved in the SM ($d = 4$) Lagrangian. The only observed lepton-flavour changing transitions are in neutrino oscillations, whereas lepton-flavour violating (LFV) transitions of charged leptons are severely constrained by experiments.

Before analysing the efficiency of the transformation (45) in suppressing LFV processes, it is worth stressing that it has a non-trivial impact also in quark FCNC transitions. Indeed four-fermion operators with a leptonic current, such as

$$\bar{Q}_L^i \gamma^\mu Q_L^j \bar{L}_L^k \gamma_\mu L_L^\ell, \quad \bar{D}_R^i Q_L^j \bar{L}_L^k E_R^\ell, \quad (46)$$

receive lepton suppression factors in addition to those from the quark wave functions (the coefficients are $\sim z_Q^{(i)} z_Q^{(j)} z_L^{(k)} z_L^{(\ell)}$ and $\sim z_Q^{(j)} z_D^{(i)} z_L^{(k)} z_E^{(\ell)}$, respectively). This implies that such operators are totally negligible. Their contributions to lepton-flavour conserving processes, such as $B(K) \rightarrow \ell^+ \ell^-$ or $B(K) \rightarrow \pi \ell^+ \ell^-$, are well below the size expected in the MFV framework. Similarly, their contributions to neutral-current processes which violate both quark and lepton flavour, such as $B_0 \rightarrow \tau \bar{\mu}$ or $K_L^0 \rightarrow \mu \bar{e}$, are well below the current experimental sensitivity.

4.1. Bounds from LFV processes

In the lepton sector the most stringent constraints are on $\Delta F = 1$ transitions among the first two generations ($\mu \rightarrow e\gamma$, $\mu \rightarrow e\bar{e}e$ and $\mu \rightarrow e$ conversion on nuclei). $\Delta F = 2$ processes, such as muonium–anti-muonium conversion [34], are less restrictive. In our scenario the largest rates for $\Delta F = 1$ processes arise from higher-dimensional operators bilinear in the lepton fields (suppressed only by two $z_{L,E}$ factors) which induce $\mu e\gamma$ and μeZ effective interactions. So we focus on electroweak dipole operators such as

$$O_{RL1}^{ij} = g' H^\dagger \bar{E}_R^i \sigma^{\mu\nu} L_L^j B_{\mu\nu}, \quad O_{RL2}^{ij} = g H^\dagger \bar{E}_R^i \sigma^{\mu\nu} \tau^a L_L^j W_{\mu\nu}^a \quad (47)$$

and operators contributing to the $\ell\ell'Z$ vertex⁴

$$O_{LL}^{ij} = \bar{L}_L^i \gamma^\mu L_L^j H^\dagger D_\mu H, \quad O_{RR}^{ij} = \bar{E}_R^i \gamma^\mu E_R^j H^\dagger D_\mu H. \quad (48)$$

We start analysing the constraints from the radiative LFV decays, which are sensitive to dipole operators only and which turn out to be the most significant processes. Introducing the effective Lagrangian

$$\mathcal{L} = \frac{1}{\Lambda^2} \sum X_{RLX}^{ij} O_{RLX}^{ij} + \text{h.c.}, \quad (49)$$

the radiative branching ratios can be written as⁵ [33]

$$\mathcal{B}(l_i \rightarrow l_j \gamma) = \frac{\Gamma(l_i \rightarrow l_j \gamma)}{\Gamma(l_i \rightarrow l_j \nu \bar{\nu})} \mathcal{B}(l_i \rightarrow l_j \nu \bar{\nu})$$

$$= \frac{192 \pi^3 \alpha}{G_F^2 \Lambda^4} \frac{1}{(\lambda_E)^2_{ii}} [|X_{RL\gamma}^{ij}|^2 + |X_{RL\gamma}^{ji}|^2] b_{ij}, \quad (50)$$

where $X_{RL\gamma}^{ij} = X_{RL2}^{ij} - X_{RL1}^{ij}$ and $b_{ij} = \mathcal{B}(l_i \rightarrow l_j \nu \bar{\nu})$, $\{b_{\mu e}, b_{\tau e}, b_{\tau \mu}\} = \{1.0, 0.178, 0.173\}$. We can already see that these branching ratios tend to be large in our scenario: the $z_L^{(i)} z_E^{(j)}$ suppression is partially compensated by the $1/(\lambda_E)^2_{ii}$ term (which appears because of the normalization to $\Gamma(l_i \rightarrow l_j \nu \bar{\nu}) \propto m_i^5$) and if the new physics gives $l_i \rightarrow l_j \gamma$ via loops, the associated $1/16\pi$ (which is absorbed in Λ^2) is compensated by the three body final state phase space of $l_i \rightarrow l_j \nu \bar{\nu}$.

The decomposition $X_{RL}^{ij} \sim z_E^{(i)} z_L^{(j)}$ leads to

$$\mathcal{B}(\mu \rightarrow e \gamma) \approx 1.2 \times 10^{-11} \left(\frac{130 \text{ TeV}}{\Lambda} \right)^4$$

$$\times \left(\frac{|z_L^{(1)} z_E^{(2)}|^2}{2(\lambda_E)_{22}(\lambda_E)_{11}} + \frac{|z_E^{(1)} z_L^{(2)}|^2}{2(\lambda_E)_{22}(\lambda_E)_{11}} \right), \quad (51)$$

$$\mathcal{B}(\tau \rightarrow e \gamma) \approx 1.1 \times 10^{-7} \left(\frac{4.3 \text{ TeV}}{\Lambda} \right)^4$$

$$\times \left(\frac{|z_L^{(1)} z_E^{(3)}|^2}{2(\lambda_E)_{11}(\lambda_E)_{33}} + \frac{|z_E^{(3)} z_L^{(1)}|^2}{2(\lambda_E)_{33}(\lambda_E)_{11}} \right), \quad (52)$$

$$\mathcal{B}(\tau \rightarrow \mu \gamma) \approx 4.5 \times 10^{-8} \left(\frac{20 \text{ TeV}}{\Lambda} \right)^4$$

$$\times \left(\frac{|z_L^{(2)} z_E^{(3)}|^2}{2(\lambda_E)_{22}(\lambda_E)_{33}} + \frac{|z_E^{(3)} z_L^{(2)}|^2}{2(\lambda_E)_{33}(\lambda_E)_{22}} \right), \quad (53)$$

where the scale for each mode has been chosen such that the branching ratio is close to its current experimental bound [35–37].

Using a CKM-type ansatz ($z_L^{(3)} \sim 1$, $z_L^{(2)} \sim \lambda^2$, $z_L^{(1)} \sim \lambda^3$, with $\lambda \sim 0.2$), for $\Lambda < 10$ TeV both $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e \gamma$ are within their experimental bounds. On the contrary, $\mu \rightarrow e \gamma$ exceeds its bound by six (!) orders of magnitude. The problem of $\mu \rightarrow e \gamma$ persists with any reasonable ansatz (such as the “democratic” assignment $z_L^{(i)}/z_L^{(j)} \sim z_E^{(i)}/z_E^{(j)} \sim (m_i/m_j)^{1/2}$). We thus conclude that either the scale of the LFV operators is pushed well above 10 TeV or, equivalently, the corresponding couplings are suppressed by several orders of magnitude compared to their naive expectation in this framework.⁶

Similar (slightly less stringent) bounds on the dipole operators are obtained from their contributions to $\mu \rightarrow e$ conversion in nuclei and $\mu \rightarrow 3e$. We finally briefly consider the LL and RR operators in Eq. (48). After electroweak symmetry breaking,

⁵ We have neglected the helicity-suppressed interference term between $X_{RL\gamma}^{ij}$ and $X_{RL\gamma}^{ji}$.

⁶ For instance, if the dipole operator is generated only via an effective four-lepton interaction (with two lepton lines closed into a loop), its coupling receives an extra suppression factor which allow to set $\Lambda \sim 10$ TeV. Similarly, dipole operators are dynamically suppressed in the RS scenario considered in Ref. [14], where the new degrees of freedom are only vector-like.

⁴ The complete basis of LFV operators can be found for instance in Ref. [32]

Table 1
Comparison of the parametrical suppression factors of the various quark bilinears, both in the MFV framework and in the general scenario with hierarchical kinetic terms

| Quark bilinears | MFV | | Hierarchical kinetic terms | |
|-----------------|---|----------------------|---|-------------------------|
| | Parametric size | Comparison with exp. | Parametric size | Comparison with exp. |
| $L^i L^j$ | $V_{ti}^* V_{tj}$ | close to experiment | $V_{ti}^* V_{tj}$ | same size as MFV |
| $L^i R^j$ | $(\lambda_D)_{ii} V_{ti}^* V_{tj}$ | negligible | $(\lambda_D)_{ii} \frac{V_{tj}}{V_{ti}}$ | can exceed exp. bounds |
| $R^i R^j$ | $(\lambda_D)_{ii} V_{ti}^* V_{tj} (\lambda_D)_{jj}$ | negligible | $\frac{(\lambda_D)_{ii} (\lambda_D)_{jj}}{V_{ti}^* V_{tj}}$ | comparable to $L^i L^j$ |

and integrating out the heavy Z field, these give rise to four-fermion operators of the type $\bar{f}^k \gamma^\mu f^k \bar{L}_L^i \gamma_\mu L_L^j$, where the f^k can be any of the SM fermions. These contribute to $\ell_i \rightarrow 3\ell$ decays, $\mu \rightarrow e$ conversion, and other previously discussed quark and lepton operators. For $\Lambda \sim 10$ TeV, and with “democratic” assignment $X_{LL,RR}^{ij} \sim \sqrt{(\lambda_E)_{ii} (\lambda_E)_{jj}}$, the expected rate are all below the experimental bounds.

5. Discussion and comparison to MFV

The analysis of the previous sections can be summarized as follows. In the quark sector the hierarchy of the fermion kinetic terms necessary to naturally reproduce both Yukawa structures and suppress dimension-six LL operators is

$$\begin{aligned} z_Q^{(i)} &\sim V_{ti} \sim \mathcal{O}(1), \mathcal{O}(\lambda^2), \mathcal{O}(\lambda^3), \\ z_D^{(i)} &\sim \frac{(\lambda_D)_{ii}}{V_{ti}} \sim \mathcal{O}(\lambda^2), \mathcal{O}(\lambda^3), \mathcal{O}(\lambda^4). \end{aligned} \quad (54)$$

Employing this hierarchy, most of the predictions from other $\Delta F = 2$ and $\Delta F = 1$ dimension-six FCNC operators fall within the experimental bounds without fine tuning, i.e. assuming generic $\mathcal{O}(1)$ couplings in the basis where the kinetic terms are hierarchical, and an effective scale of new physics ~ 10 TeV. The only exceptions are the LR operators contributing to ϵ_K and ϵ'/ϵ , which can be sufficiently suppressed with a modest fine tuning of the hierarchies and $\mathcal{O}(1)$ coefficients.⁷ This scenario predicts that new physics in ϵ_K and ϵ'/ϵ is just around the corner: a conspiracy to suppress the LR operator coefficients by another order of magnitude would require a too severe fine tuning (it occurs with a probability $\lesssim 0.5\%$).

Given the consistency of this general framework in suppressing quark FCNC amplitudes, it is useful to compare it with the more precise predictions of the MFV framework (from which the fuzzy $\mathcal{O}(1)$ factors are absent). In Table 1 we list quark bilinears and their suppression factors, as expected within MFV and within the framework with hierarchical fermion profiles. The differences arise for RR and LR operators. Within the MFV hypothesis these are strongly suppressed by one or two powers of the down-type Yukawa coupling. However, such as suppression is not necessary for the description of nature (especially in the case of RR operators) and is partially removed in the scenario with hierarchical fermion profiles. Note that, while in the MFV framework is possible to enhance the overall normalization of the down-type Yukawa couplings, considering two Higgs fields and a large vev ratio ($\tan\beta = \langle H_U \rangle / \langle H_D \rangle \gg 1$), this is not possible in the scenario with hierarchical kinetic terms. In the latter case a possible $\tan\beta$ enhancement of the Yukawa couplings would produce a strong tension with data in the LR sector.

The situation of the lepton sector is more problematic. The constraints on helicity conserving (LL and RR) LFV operators are satisfied assuming an hierarchy of the type (54) for $z_L^{(i)}$ and $z_E^{(i)}$ (with $\lambda_D \rightarrow \lambda_E$) or, equivalently, the democratic assignment $z_{L,E}^{(i)} \sim \sqrt{(\lambda_E)_{ii}}$. However, the constraints on LR operators contributing to $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion, require an effective scale in the 100 TeV range. It is therefore difficult to make predictions: if the LFV rates are suppressed because the new physics scale Λ is high in the lepton sector, then $\mu \rightarrow e\gamma$ could be close to its present exclusion bound, $\mathcal{B}(\mu \rightarrow 3e)$ and the rate for $\mu \rightarrow e$ conversion are suppressed by $\mathcal{O}(\alpha)$ with respect to $\mathcal{B}(\mu \rightarrow e\gamma)$, and τ LFV decays are beyond the reach of future facilities. However, these predictions do not hold if LFV dipole operators are suppressed by some specific dynamical mechanism. In the latter case we cannot exclude scenarios where the $\tau \rightarrow \mu\gamma$ rate is close to its present exclusion bound.

Given the important role of dipole operators in this framework, it is worth to look at the bounds derived from the corresponding flavour-diagonal partners contributing to anomalous-magnetic and electric-dipole moments, both in the quark and in the lepton sector. As far as anomalous-magnetic moments are concerned, the most significant constraint comes from $(g-2)_\mu$. Here we could solve the current discrepancy [38] only for $\Lambda \sim 2-3$ TeV (setting $z_E^{(2)} z_L^{(2)} \sim (\lambda_E)_{22}$), a scale which is far too low compared to the $\mu \rightarrow e\gamma$ bound. We thus conclude that there is no significant contribution to $(g-2)_\mu$ in this framework. On the other hand, stringent bounds on Λ (in the ~ 100 TeV range) are imposed by the electron and the neutron electric-dipole moments (assuming $z_{E(D)}^{(1)} z_{L(Q)}^{(1)} \sim (\lambda_{E(D)})_{11}$ and $\mathcal{O}(1)$ flavour-diagonal CP-violating phases). Being flavour-conserving and CP-violating, the coupling of these operators could easily be suppressed by independent mechanisms, such as an approximate CP invariance in the flavour-conserving part of the Lagrangian. However, the fact that these bounds are close to those derived from $\mu \rightarrow e\gamma$ can also be interpreted as a further indication of a common dynamical suppression mechanism of dipole-type operators.

6. Conclusions

The absence of deviations from the SM in the flavour sector points toward extensions of the model with highly non-generic flavour structures. In this Letter we have investigated the viability of models where the suppression of flavour-changing transitions is not attributed to a specific flavour symmetry, but it arises only from appropriate hierarchies in the kinetic terms. A generic framework which could occur in models with extra dimensions.

We have considered in particular the class of scenarios where the Yukawa matrices and the dimensionless flavour-changing couplings of the higher-dimensional operators do not exhibit a specific flavour structure (i.e. have generic $\mathcal{O}(1)$ entries) in a basis where the kinetic terms are hierarchical. Despite its simplicity, this construction is sufficient to suppress to a level consistent with experiments all the flavour-changing transitions in the quark sector, assuming a scale of new physics in the TeV range. The only two ob-

⁷ To estimate the amount of fine tuning, we assume the $\mathcal{O}(1)$ coefficients c to be distributed as $P(|c|) \propto \exp[-(\ln|c|)^2/2]$. Under this assumption the coefficients of the FCNC operators are obtained via a maximum likelihood analysis and we find a $\sim 15\%$ probability to be consistent with the experimental bounds.

servables where a mild tuning of the $\mathcal{O}(1)$ coefficient is needed are the CP-violating parameters of the neutral kaon system: within this scenarios new physics effects in ϵ_K and ϵ'/ϵ should be detectable with improved control on the corresponding SM predictions.

The most serious challenge to this class of models appears in the lepton sector, thanks to the stringent bounds on $\mu \rightarrow e$ transitions. The latter require either an heavy effective scale of LFV ($\Lambda_{\text{LFV}} \gtrsim 100$ TeV) or an independent dynamical suppression mechanism for dipole-type operators.

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